All solutions to graded problems must be typed in LaTeX. All other problems can be typed or written, but must be completed. Graded problems must be organized in numerical order. All problem sets (both graded and non-graded) must be uploaded to Canvas as a single PDF document. Students can find the grading scale in our Syllabus.

Graded

Exercise 1. Let $f : A \to B$ be given. Let $A_0 \subseteq A$ and $B_0 \subseteq B$.

1. Show that $A_0 \subseteq f^{-1}(f(A_0))$ and that equality holds if f is injective.
2. Show that $f(f^{-1}(B_0)) \subseteq B_0$ and that equality holds if f is surjective.

Exercise 2. Let $f : A \to B$ and $g : B \to C$.

1. If $C_0 \subset C$, show that $(g \circ f)^{-1}(C_0) = f^{-1}(g^{-1}(C_0))$.
2. If f and g are injective, show that $g \circ f$ is injective.
3. If $g \circ f$ is injective, what can you say about injectivity of f and g? Prove your claims.
4. If f and g are surjective, show that $g \circ f$ is surjective.
5. If $g \circ f$ is surjective, what can be said about surjectivity for f and g? Prove your claims.

Non-graded

Exercise 3. Let $f : A \to B$, let $i \in \{1, 2\}$, and let $A_i \subseteq A$ and $B_i \subseteq B$. Show that f^{-1} (preimages) preserves inclusions, unions, intersections, and differences of sets.

1. $B_0 \subseteq B_1 \Rightarrow f^{-1}(B_0) \subseteq f^{-1}(B_1)$
2. $f^{-1}(B_0 \cup B_1) = f^{-1}(B_0) \cup f^{-1}(B_1)$
3. $f^{-1}(B_0 \cap B_1) = f^{-1}(B_0) \cap f^{-1}(B_1)$
4. $f^{-1}(B_0 - B_1) = f^{-1}(B_0) - f^{-1}(B_1)$

Show that f preserves only inclusions and unions.

5. $A_0 \subseteq A_1 \Rightarrow f(A_0) \subseteq f(A_1)$
6. $f(A_0 \cup A_1) = f(A_0) \cup f(A_1)$
7. $f(A_0 \cap A_1) \subseteq f(A_0) \cap f(A_1)$; with equality if f is injective.
8. $f(A_0 - A_1) \supseteq f(A_0) - f(A_1)$; with equality if f is injective.

Exercise 4. Show that the equalities and inclusions from Exercise 3 parts 2, 3, 6, and 7 hold for arbitrary unions and intersections.

Exercise 5. Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^3 - x$. Find a restriction of both the domain $A \subset \mathbb{R}$ and codomain $B \subset \mathbb{R}$ so that the function $g : A \to B$ defined by $g(a) = f(a)$ for all $a \in A$ is bijective. Draw the graphs of g and g^{-1}. *Hint: There are more than one possible g.*